

# Skill Builders

(Extra Practice)

## Volume I

1. Factoring Out Monomial Terms
2. Laws of Exponents
3. Function Notation
4. Properties of Lines
5. Multiplying Binomials
6. Special Triangles
7. Simplifying and Combining Radicals
8. Similar Triangles
9. Factoring Standard Forms
10. Inverses
11. Algebraic Fractions
12. Solving Quadratics
13. Inequalities

## 1. Factoring Out Monomial Terms

**Example:** The first thing you should look for when factoring expressions is common factors that appear in every term. These common factors could be numbers or variables, so look for common numerical factors and common variables that appear in every term.

$$3x^2y^5 - 5y^3 = 3xy^3(xy^2 - 2)$$

Factor out the largest single term possible from the following expressions.

1.  $2x^3y + 7xy^2$

2.  $42a^3b^3 + 24a^2b^5$

3.  $24\pi r^4h - 20\pi r^2h^4$

4.  $30b^{11}c^6d^4 + 15b^{11}c^7d^3 - 35b^{10}c^7d^4$

5.  $36x^5y^5z^6 - 27x^3y^3z^{10} + 18x^6y^2z^7$

6.  $6a^4b^2 - 7ab^4 + 7a^2b^2$

7.  $27p^6q + 45p^5q^2 - 54p^8q$

8.  $8j^{10}k^2 + 20j^3k^4$

9.  $21x^4y^3 + 35x^2y^8$

10.  $42d^3h^3 + 63d^2h^5$

11.  $24\pi r^2h^2 + 18\pi r^2h + 54\pi r^2$

12.  $15mn^4 - 33m^3n$

13.  $2a^5b^5c - 6a^3b^3c^6 + 7a^2b^6c^3$

14.  $15s^2t^5 - 25st^7 + 55st^5$

15.  $8u^3v - 32uv^4 + 80uv$

16.  $6x^4y^3 + 8x^2y^5 - 2x^3y^2 + 6x^2y^2$

## 2. Laws of Exponents

**Example:** Simplify using the laws of exponents.

**Solution:**

$$\left(\frac{2x^{-3}y^2}{y^3}\right)^{-4} = 2^{-4}(x^{-3-1})(y^{2-3})^{-4} = 2^{-4}x^{-4}y^{-4} = \frac{x^4y^4}{16}$$

Simplify the following expressions.

1.  $(x^3)(x^4)$

2.  $(y^2)(y^3)$

3.  $\frac{a^9}{a^6}$

4.  $\frac{p^7}{p^2}$

5.  $(x^3)^3$

6.  $(y^2)^5$

7.  $[(z^2)^3]^2$

8.  $[(b^3)^3]^3$

9.  $[(z^2)(x^4)]^3$

10.  $[(y^{-2})(y^3)]^0$

11.  $[(a^{-3})(a^5)]^3$

12.  $\left(\frac{x^4}{x}\right)^2$

13.  $(x^{-3})(x^3) + (y^4)(y^{-4})$

14.  $(2x)^3(3x)^2$

15.  $\frac{(5x)^2(2x)^2}{5x}$

16.  $\left[\frac{(3x)^3(2x)^2}{4x}\right]^2$

### 3. Function Notation

**Example:** Let  $f(x) = 2x^2 - 2x + 3$  and  $g(x) = 3x - 5$ . Find the indicated values.

**Solutions:**

a.  $f(-2) = 2(-2)^2 - 2(-2) + 3 = 15$

b.  $f(x+h) = 2(x+h)^2 - 2(x+h) + 3$   
 $= 2(x^2 + 2xh + h^2) - 2(x+h) + 3$   
 $= 2x^2 + 4xh + 2h^2 - 2x - 2h + 3$

c.  $g(f(x)) = g(2x^2 - 2x + 3) = 3(2x^2 - 2x + 3) - 5$   
 $= 6x^2 - 6x + 4$

Find the indicated values.

1. Given  $f(x) = 3x^2 + 5$ , find  $f(4)$ .
2. Given  $f(x) = 5x^3 + 3x^2$ , find  $f(-3)$ .
3. Given  $f(x) = 5x^2 + 3x$ , find  $f(x+h)$ .
4. Given  $f(x) = 21x^2 - 20x$ , find  $f(x+h)$ .
5. Given  $f(x) = 3x^2 + 5x$  and  $g(x) = 7x + 3$ , find  $f(x)g(x)$ .
6. Given  $f(x) = 6x^3 + 7$  and  $g(x) = e^x$ , find  $f(x)g(x)$ .
7. Given  $f(x) = 3x^2 + 4x + 5$  and  $g(x) = x^2 + x + 1$ , find  $f(x)g(x)$  if  $x = 3$ .
8. Given  $f(x) = 2x^2 + 5x$  and  $g(x) = 3x$ , find  $f(g(x))$ .
9. Given  $f(x) = 2x^2 + 5x$  and  $g(x) = 3x$ , find  $g(f(x))$ .
10. Given  $g(x) = 5x^2 + 5x + 5$  and  $h(x) = 4x + 4$ , find  $g(h(x))$ .
11. Given  $g(x) = 5x^2 + 5x + 5$  and  $h(x) = 4x + 4$ , find  $h(g(x))$ .
12. Given  $h(x) = 16 \cdot 3^{5x}$ , find  $h(2)$ .
13. Given  $f(x) = 5 \cdot 2^{3x}$ , find  $f(3)$ .
14. Given  $f(x) = e^x$  and  $g(x) = 3x^2 + 2x + 6$ , find  $f(g(x))$ .
15. Given  $f(x) = e^x$  and  $g(x) = 3x^2 + 2x + 6$ , find  $g(f(x))$ .

## 4. Properties of Lines

Given the points  $A = (-2, 5)$  and  $B = (3, -4)$ .

**Example 1:** Find the equation of the line that goes through  $A$  and  $B$ . Write your answer in slope-intercept form.

**Solution:**  $\text{slope} = \frac{(5 - (-4))}{(-2 - 3)} = \frac{9}{-5} = -\frac{9}{5}$

Now use the form  $y = mx + b$  with  $m = -\frac{9}{5}$  to get the equation  $y = -\frac{9}{5}x + b$ . Substitute the  $x$  and  $y$  from point  $A$  or point  $B$  and solve for  $b$ .

$$5 = -\frac{9}{5}(-2) + b$$

$$5 = \frac{18}{5} + b$$

$$b = \frac{7}{5}$$

$$y = -\frac{9}{5}x + \frac{7}{5}$$

**Example 2:** Find the equation of the line that is perpendicular to line  $AB$  and passes through the point  $(2, 3)$ .

**Solution:** Perpendicular lines have negative reciprocal slopes. Since the slope of  $AB$  is  $-\frac{9}{5}$ , the slope of the line perpendicular to it is  $\frac{5}{9}$ . Point-slope equations have the form  $y - y_1 = m(x - x_1)$ . Substitute  $(2, 3)$  for  $(x_1, y_1)$  and  $\frac{5}{9}$  for  $m$  to get:  $y - 3 = \frac{5}{9}(x - 2)$ .

**Example 3:** Find the distance between points  $A$  and  $B$ .

**Solution:**  $d = \sqrt{(-2 - 3)^2 + (5 - (-4))^2} = \sqrt{(-5)^2 + (9)^2} = \sqrt{106}$

1. Find the equation of the line that is parallel to the line  $y = 3x + 9$  and passes through the point  $(-4, 3)$ . Write your answer in point-slope form.
2. Find the equation of the line that is parallel to the line  $y = 2x + 8$  and passes through the point  $(-4, 3)$ . Write your answer in slope-intercept form.
3. Find the equation of the line that is parallel to the line  $y = -6x - 8$  and passes through the point  $(8, 3)$ . Write your answer in point-slope form.
4. Find the equation of the line that is parallel to the line  $y = -5x - 9$  and passes through the point  $(7, 1)$ . Write your answer in slope-intercept form.
5. Find the equation of the line that contains the points  $(1, 7)$  and  $(-5, -3)$ . Write your answer in slope-intercept form.
6. Find the equation of the line that contains the points  $(-5, -10)$  and  $(9, 6)$ . Write your answer in slope-intercept form.
7. Find the equation of the line that contains the points  $(-1, 4)$  and  $(-5, -4)$ . Write your answer in point-slope form.
8. Find the equation of the line that contains the points  $(5, 2)$  and  $(8, 11)$ . Write your answer in point-slope form.
9. Find the equation of the line that is perpendicular to the line  $y = 7x + 1$  and passes through the point  $(3, -1)$ . Write your answer in point-slope form.
10. Find the equation of the line that is perpendicular to the line  $y = 6x + 2$  and passes through the point  $(3, 7)$ . Write your answer in slope-intercept form.
11. Find the equation of the line that is perpendicular to the line  $y = -9x - 8$  and passes through the point  $(1, 6)$ . Write your answer in slope-intercept form.
12. Find the equation of the line that is perpendicular to the line  $y = 2x + 5$  and passes through the point  $(-6, -7)$ . Write your answer in slope-intercept form.
13. Find the distance between the points  $(-3, -5)$  and  $(5, 4)$ .
14. Find the distance between the points  $(-1, 6)$  and  $(3, -4)$ .
15. Find the distance between the points  $(4, 3)$  and  $(18, 14)$ .
16. Find the distance between the points  $(0, 7)$  and  $(4, 16)$ .

## 5. Multiplying Binomials

**Example:** Multiply  $(x + 2)(x - 5)$ . Each term in the first binomial is distributed over the second binomial, as shown below.

**Solution:**  $(x + 2)(x - 5) = x(x - 5) + 2(x - 5) = x^2 - 5x + 2x - 10 = x^2 - 3x - 10$

Multiply the following binomials.

1.  $(x + 3)(x - 6)$

2.  $(a^3 + 1)(a^2 - 4)$

3.  $(b^2 - 3)(b + 5)$

4.  $(z^3 + 1)(z^2 - 1)$

5.  $(p^{11} - p^{10})(p^{10} - p^9)$

6.  $(q^4 + 4)(q^3 + 3)$

7.  $(5h^4 + 1)(6h - 2)$

8.  $(d^5 - d^4)(d^6 + 1)$

9.  $(x + 3)(x + 6)(x - 7)$

10.  $(x^2 + 3)(x - 2)(x + 1)$

11.  $(m^{99} + m^{98})(m^{97} - m^{96})$

12.  $(6n^4 + 3n^3)(6n^3 + 3n^2)$

13.  $(100j^{100} - j)(9j^2 + 1)$

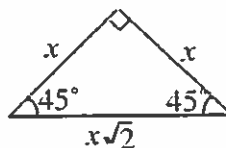
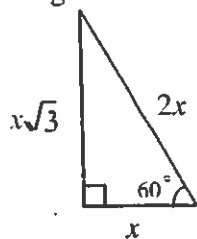
14.  $(2\pi k^5 + 5k^4)(\pi k + 3)$

15.  $(c^2x + d^3y)(2x + c^2d)$

16.  $(f^3y^3 + g^2x^2)(fy + gx)$

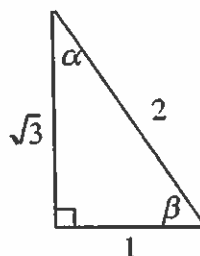
## 6. Special Triangles

Example: The following relationships exist between the sides of the special triangles.



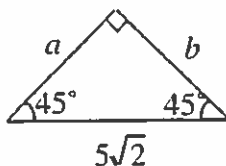
Use these relationships to solve each of the following problems.

1. Solve for angles  $\alpha$  and  $\beta$  in the triangle at right.



2. Given a 30-60-90 triangle with hypotenuse length 8, find the exact length of the long leg.
3. A 30-60-90 triangle has a hypotenuse 0.5 inches long. How long is the short leg?
4. Given a 30-60-90 triangle whose long leg is 13 cm, how long is the hypotenuse? (Solve exactly.)

5. Solve for leg lengths  $a$  and  $b$ .



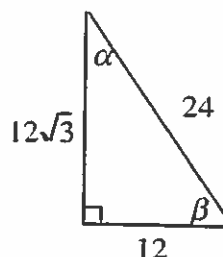
6. Given a 45-45-90 triangle with hypotenuse length  $13\sqrt{2}$ , how long are the legs?
7. If a 45-45-90 triangle's legs are 17 mm long, exactly how long is its hypotenuse?
8. What is the exact area of the equilateral triangle whose legs are each 6 cm long?



9. Find the exact area of the following isosceles triangle.



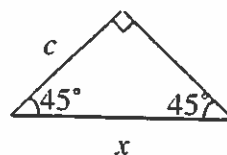
10. A triangle has angles  $30^\circ$ ,  $60^\circ$ , and  $90^\circ$ , and its legs have lengths  $7\sqrt{3}$ , 7, and 14. How long is the side opposite the  $60^\circ$  angle? [ $7\sqrt{3}$ ]
11. Solve for  $\alpha$  and  $\beta$  in the following triangle.



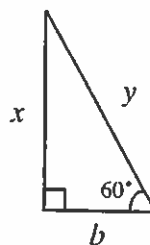
12. If a 30-60-90 triangle has a hypotenuse length 100, how long is the short leg?
13. A 30-60-90 triangle with long leg length  $15\sqrt{3}$  has a hypotenuse of what length?
14. Find the area of the triangle at right.



15. Solve for length  $x$  in terms of  $c$ .



16. Solve for lengths  $x$  and  $y$  in terms of  $b$ .



## 7. Simplifying and Combining Radicals

Example: Simplify  $\sqrt{288}$ .

When you are asked to write a radical in simpler form, the real task is to reduce the number under the radical sign as much as possible by taking perfect squares outside the radical.

*Solution:* Simplify  $\sqrt{288}$ . Find the factorization of 288, choosing perfect squares as factors whenever possible.

$$288 = 4 \cdot 4 \cdot 9 \cdot 2$$

$$\sqrt{288} = \sqrt{4} \sqrt{4} \sqrt{9} \sqrt{2}$$

$$\sqrt{288} = 2 \cdot 2 \cdot 3 \cdot \sqrt{2}$$

$$\sqrt{288} = 12\sqrt{2}$$

Rewrite the following as radicals in simpler form.

1.  $\sqrt{1200}$

2.  $\sqrt{845}$

3.  $\sqrt{a^3b^5}$

4.  $\sqrt{x^7b^9}$

Example: Simplify  $3\sqrt{125} - 2\sqrt{80}$ .

*Solution:* First simplify each radical separately and then combine like terms.

$$3\sqrt{125} = 3\sqrt{25}\sqrt{5} = 15\sqrt{5}$$

$$2\sqrt{80} = 2\sqrt{16}\sqrt{5} = 8\sqrt{5}$$

$$3\sqrt{125} - 2\sqrt{80} = 15\sqrt{5} - 8\sqrt{5} = 7\sqrt{5}$$

Simplify the following expressions.

5.  $\sqrt{5} + \sqrt{18}$

6.  $\sqrt{54} + \sqrt{294}$

7.  $\sqrt{18} + \sqrt{108} + \sqrt{50} + \sqrt{48}$

8.  $(\sqrt{99})^2$

9.  $(\sqrt{12})(\sqrt{75})$

10.  $(\sqrt{20})(\sqrt{80})$

11.  $(\sqrt{50})(\sqrt{98})$

**Example:** Rationalize  $\frac{4}{\sqrt{5}}$ .

**Solution:** To rationalize the denominator you need to multiply both numerator and denominator by the radical in the denominator. This is equivalent to multiplying by 1 so it does not change the value of the expression.

$$\frac{4}{\sqrt{5}} = \frac{4}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{4\sqrt{5}}{5}$$

Rationalize the following expressions.

12.  $\frac{3}{\sqrt{2}}$

13.  $\frac{5}{\sqrt{7}}$

14.  $\frac{a^2b^4}{\sqrt{a^5b^9}}$

**Example:** Combine and simplify  $\frac{2}{\sqrt{3}} + \frac{1}{2\sqrt{3}}$ .

**Solution:** To combine rational expressions you first need to find a common denominator and write the expression as one fraction. Then, you must rationalize the denominator.

$$\frac{2}{\sqrt{3}} + \frac{1}{2\sqrt{3}} = \frac{2 \cdot 2}{2\sqrt{3}} + \frac{1}{2\sqrt{3}} = \frac{5}{2\sqrt{3}} = \frac{5\sqrt{3}}{6}$$

Rewrite the following as single, rationalized expression.

15.  $\frac{3}{\sqrt{2}} + \frac{6}{\sqrt{2}}$

16.  $\frac{6}{\sqrt{8}} + \frac{3}{\sqrt{8}}$

## 8. Similar Triangles

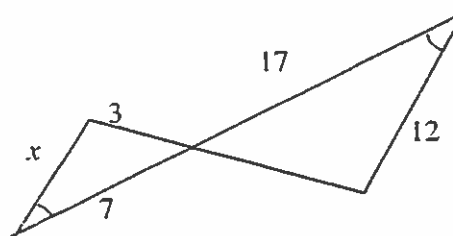
**Example:** Find the length of  $x$ .

**Solution:** To find  $x$  you need to put corresponding sides in a ratio. In this triangle 17 corresponds to 7 and 12 corresponds to  $x$ .

$$\text{So our ratio is: } \frac{x}{12} = \frac{7}{17}$$

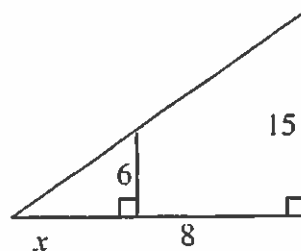
$$17x = 84$$

$$x = 4.941$$

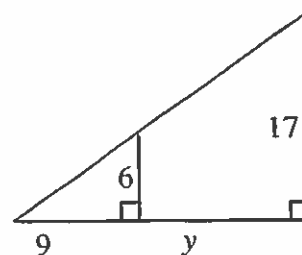


Solve each triangle for the indicated variable.

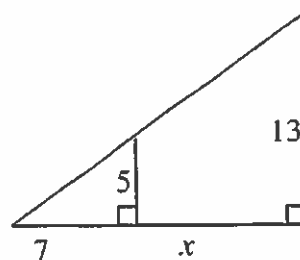
- Find the length of  $x$ .



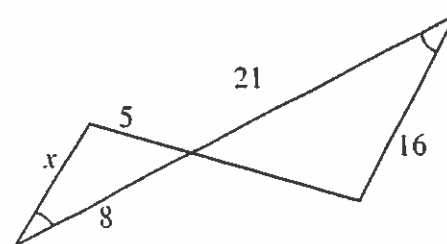
- Find the length of  $y$ .



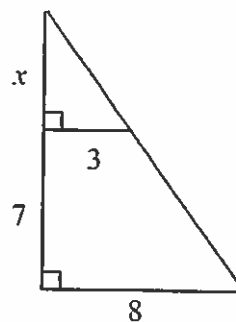
- Find the length of  $x$ .



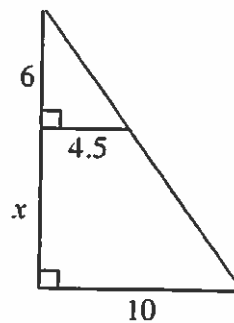
- Find the length of  $x$ .



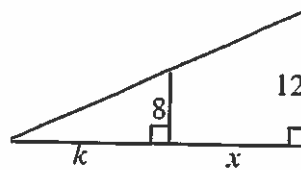
5. Find the length of  $x$ .



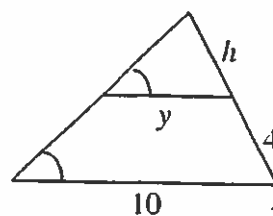
6. Find the length of  $x$ .



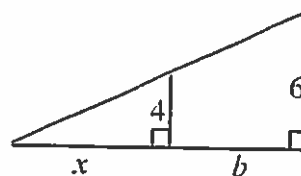
7. Find the length of  $k$  in terms of  $x$ .



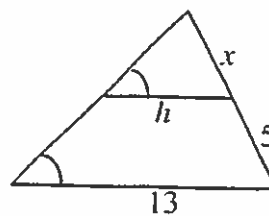
8. Find the value of  $h$  in terms of  $y$ .



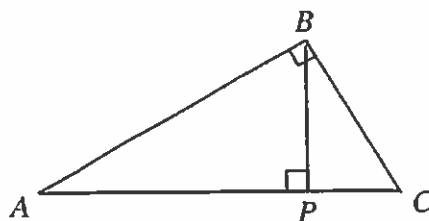
9. Find the length of  $b$  in terms of  $x$ .



10. Find the value of  $h$  in terms of  $x$ .



11. Kathleen is 6 feet tall. Her shadow is 3 feet long. At the same time of day, a tree casts a shadow that is 21 feet long. How tall is the tree?
12. Kyle is 1.5 meters tall and casts a shadow 4 meters long. A flagpole casts a shadow that is 30 meters long. How tall is the flagpole?
13. Lisa's dog is 2.5 feet tall and casts a shadow that is 4 feet long. A fire hydrant casts a shadow that is 6 feet long. How tall is the fire hydrant?
14. Fernando is constructing a scale model of his father's yacht. The actual yacht has a sail that is 25 feet tall and the base of the sail is 15 feet long. If the model's sail is 14 inches tall, how long is its base?
15. Cameron is building a scale model of his sailboat. The right-triangular sail of the actual boat has a base that is 12 feet long. The height of the actual sail is 20 feet long. The base of the model's sail is 8 inches long. How long is the hypotenuse of the model's sail? (The hypotenuse is different from the height!)
16. List all of the similar triangles that are in the figure at right.



## 9. Factoring Standard Forms

### Rules for Factoring

Difference of Two Squares:  $a^2 - b^2 = (a + b)(a - b)$

Sum of Two Cubes:  $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Difference of Two Cubes:  $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

### Special Products

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

Factor each of the following expressions.

1.  $x^2 - 12x + 36$

2.  $z^2 - 6z + 9$

3.  $9a^2 + 24a + 16$

4.  $16x^2 + 40xy + 25y^2$

5.  $(x + y)^2 - 4(x + y) + 4$

6.  $4(a - b)^2 - 8(a - b) + 4$

7.  $b^2 - 16$

8.  $7x^4 - 7$

9.  $4x^2 - 1$

10.  $(a - b)^2 - (a + b)^2$

11.  $s^6 - t^6$

12.  $x^3 + 1$

13.  $a^3 - b^3$

14.  $8p^3 + 27q^3$

## 10. Inverses

**Example:** Find the inverse of  $f(x) = \frac{x-2}{x}$

**Solution:**

$$y = \frac{x-2}{x}$$

Start by replacing  $f(x)$  with  $y$ .

$$x = \frac{y-2}{2}$$

Interchange  $x$  and  $y$  in the equation.

$$xy = y - 2$$

Solve for  $y$ .

$$xy - y = 2$$

$$y(x - 1) = 2$$

$$y = \frac{2}{x-1}$$

$$f^{-1}(x) = \frac{2}{x-1}$$

Replace  $y$  with  $f^{-1}(x)$ .

Find the equation of the inverse for each function below.

1.  $f(x) = 3x + 1$

2.  $f(x) = 5 - 6x$

3.  $f(x) = x^2 + 5$

4.  $f(x) = 4x^2$

5.  $f(x) = \frac{x-4}{5}$

6.  $f(x) = \frac{x+10}{11}$

7.  $f(x) = \frac{3}{4}x + 8$

8.  $f(x) = 7 - x^2$

9.  $f(x) = 3^x$

10.  $f(x) = \log x$

11.  $f(x) = \frac{x+3}{x}$

12.  $f(x) = \frac{x-4}{x}$

13.  $f(x) = \frac{11x-12x^2}{x}$

14.  $f(x) = \frac{3x+4}{5x}$

15.  $f(x) = \frac{10x-15x^2}{5x^2}$

16.  $f(x) = \frac{7x-4}{3x}$



## 11. Algebraic Fractions

### Facts about Fractions

$$\frac{PK}{QK} = \frac{P}{Q} \text{ where } Q \neq 0 \text{ and } K \neq 0$$

$$\text{Multiplication: } \frac{P}{Q} \cdot \frac{R}{S} = \frac{PR}{QS}$$

$$\text{Division: } \frac{P}{Q} \div \frac{R}{S} = \frac{P}{Q} \cdot \frac{S}{R} = \frac{PS}{QR}$$

$$\text{Addition: } \frac{P}{Q} + \frac{R}{Q} = \frac{P+R}{Q}$$

$$\text{Subtraction: } \frac{P}{Q} - \frac{R}{Q} = \frac{P-R}{Q}$$

Simplify the following fractions.

1.  $\frac{1/a}{b}$

2.  $\frac{1/a}{1/b}$

3.  $\frac{x^2-9}{x-3}$

4.  $\frac{x^2-5x-6}{x^2-1}$

5.  $\frac{x^2-25}{5-x}$

6.  $\frac{a^3+b^3}{a^2-b^2}$

**Example:** Use Fraction Busters to solve  $\frac{x}{3} + \frac{5}{x} = 2$ .  $3 \cdot x \cdot \frac{x}{3} + 3 \cdot x \cdot \frac{5}{x} = 2 \cdot 3 \cdot x$

**Solution:** Multiply every term on both sides of the equation by all the terms in the denominator.  $3 \cdot x \cdot \frac{x}{3} + 3 \cdot x \cdot \frac{5}{x} = 6x$

$$x^2 + 15 = 6x$$

Now you have an equation without fractions.

$$x^2 - 6x + 15 = 0$$

$$x = 3 \pm i\sqrt{6}$$

Use the Quadratic Formula to solve.

Use Fraction Busters to solve for  $x$ .

7.  $\frac{1}{x} + \frac{3}{x^2} = 4$

9.  $\frac{x}{3} + \frac{1}{2} = \frac{x+2}{6} - \frac{2}{3}$

8.  $\frac{x}{3} - \frac{x+1}{5} = 2x + \frac{1}{5}$

10.  $\frac{2}{3x} + \frac{x+2}{5} = 5x + \frac{1}{3}$

Simplify the following expressions.

11.  $1 + \frac{1}{1 + \frac{1}{1+i}}$

13.  $\frac{1 - \frac{2}{x}}{1 - \frac{3}{x}}$

15.  $\frac{\frac{x-1}{x+1} - \frac{x+1}{x-1}}{\frac{x-1}{x+1} + \frac{x+1}{x-1}}$

12.  $\frac{1 - \frac{1}{y}}{\frac{1}{y}}$

14.  $\frac{\frac{1}{a} + \frac{2}{b}}{\frac{2}{a} + \frac{1}{b}}$

16.  $\frac{\frac{a+b}{2a^2}}{\frac{a-b}{4a}}$

## 12. Solving Quadratics

Solve the quadratic equations.

Example 1:  $4x^2 - 12x = 0$

*Solution:* This type of problem is easily factored into  $4x(x - 3) = 0$ .

Now you can get the solutions  $4x = 0$  or  $x - 3 = 0$ ,  $x = 0$  or  $x - 3 = 0$ .

Example 2:  $x^2 - 2x + 13 = 0$

*Solution:* This equation cannot be factored so you should use the Quadratic Formula.

$$x = \frac{2 \pm \sqrt{-48}}{2} = \frac{2 \pm 4i\sqrt{3}}{2} = 1 \pm 2i$$

Example 3:  $\sqrt{x+1} = \sqrt{x} + 2$

*Solution:* When there are square roots in the problem you will need to square both sides.

$$(\sqrt{x+1})^2 = (\sqrt{x} + 2)^2$$

$$x + 1 = x + 2\sqrt{x} + 4$$

$$\frac{1}{2} = \sqrt{x}$$

Since there is still a square root, move all the other terms away from the root.

$$\frac{1}{4} = x$$

Now square both sides.

$$\sqrt{0.25 + 2} = \sqrt{0.25} + 2$$

Make sure you plug the answer back into the original equation because sometimes extra solutions appear when you square equations.

$$1.5 = 1.5$$

Solve for the indicated variable.

1.  $9x^2 - 12x = 0$

2.  $2r + 1 = 15r^2$

3.  $1 - \frac{1}{x} = \frac{12}{x^2}$

4.  $x - \frac{4}{3x} = -\frac{1}{3}$

5.  $(y+6)(y-2) = -7$

6.  $\sqrt{x+2} = x-4$

7.  $(y-5)^2 = 9$

8.  $(y+3)^2 = 18$

9.  $x^4 - x^2 = 20$

10.  $x^4 + 20 = 9x^2$

11.  $\sqrt{x+5} = \sqrt{x} + 1$

12.  $\sqrt{x+4} = 2 - \sqrt{2x}$

13.  $x^2 + 5x + 11 = 0$

14.  $x^2 + 1 = 0$

15.  $x^2 - x + 3 = 0$

16.  $x^2 = -(5+x)$

### 13. Inequalities

**Example 1:** Solve:  $|3x - 5| \leq 13$ .

**Solution:** Start by writing the inequality.  $-13 \leq 3x - 5 \leq 13$

Add 5 to all three members.  $-8 \leq 3x \leq 18$

Divide all three members by 3.  $-\frac{8}{3} \leq x \leq 6$

The answer written in interval notation is  $\left[-\frac{8}{3}, 6\right]$ .

Remember that if you multiply or divide an inequality by a negative number you must reverse the inequality. For example, solve  $-2x \geq 18$ ,  $x \leq -9$ .

Dividing both sides by  $-2$  reverses the inequality.

**Example 2:** Solve:  $x^2 - x > 6$

**Solution:** As with equations, we need to have a polynomial equal to zero so that we can factor.

Subtract 6 from both sides.  $x^2 - x - 6 > 0$

Factor:  $(x - 3)(x + 2) > 0$

Find the zero's.  $x = 3, x = -2$

Test the intervals to find where the quadratic is greater than zero.



Here the interval is positive for values less than  $-2$  or greater than  $3$ . It is negative between  $-2$  and  $3$ .

Express the solution using inequalities  $x < -2$  or  $x > 3$ .

Solve the inequality. Write your answer in either interval or inequality notation.

1.  $|x| \geq -2$

2.  $|x - 3| < 7$

3.  $|x - 3| + 2 < 6$

4.  $|2x + 1| + 4 \geq 7$

5.  $x^2 + x - 6 > 0$

6.  $x^2 - x - 12 \leq 0$

7.  $6x^2 < 5x - 1$

8.  $(x - 2)(x - 3)(x - 4) > 0$

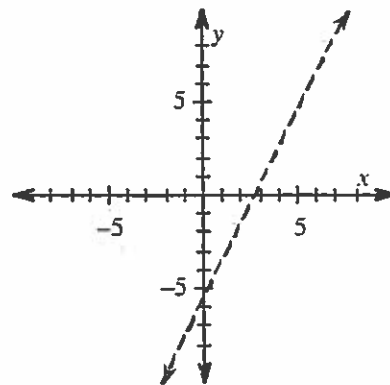
Solve the inequality and graph the solution set.

9.  $|x - 1| < -3$

10.  $|3x + 9| \geq 6$

**Example:** Graph the solution  $y + 5 > 2x$ .

**Solution:** Begin by treating the expression as an equality and sketch the graph. So begin by graphing  $y = 2x - 5$ . Since the inequality is  $y$  *greater than*  $2x - 5$  the line is dotted because it is not included. Next you should choose a test point on one side of the line and plug it into the inequality. If it makes the inequality true, then shade that side of the graph. If it makes the inequality false, then shade the other side of the graph. A good point to use (as long as it is not on the graph) is  $(0, 0)$ . If we substitute  $x = 0$  and  $y = 0$  into the inequality, then it becomes  $0 + 5 > 10$ , which is false so shade the **other** side of the line.



Graph the solution.

11.  $x - 2y < 4$

12.  $x^2 + 5x \geq -6$

13.  $2x + 3y \leq 6$

14.  $x - y > -3$

15.  $2x + y \leq 5$

16.  $3x + 5y \leq 7$